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Multicriterial analisys of selection of coal with saw and copras methods

One of the major problems in both rural and urban areas is to choose during the winter season the best alternative for heating, namely coal. In this paper we will show that the application of SAW and COPRAS methods with the appropriate criteria we can reach the best alternative. The paper presents a numerical example that shows the applicability and effectiveness of the proposed approach.

***Key words**: selection of coal, SAW, COPRAS*

1. INTRODUCTION

Approximately 30% of households in Serbia before each heating season are faced with the problem of choosing which energy source to use for heating. As the economic situation is difficult, energy prices are the largest parameter that you pay attention. It may, and may not always be the right decision. In fact there are a lot of criteria / parameters before choosing (price, calorific power, density, ash content, sulphur content...).If all the parameters are taken into account, it would probably be the best alternative and somewhat different from those obtained on the basis of just price or any other criteria. In this paper we present a numerical selection of the best alternative by applying SAW (Simple Additive Weighting) and COPRAS (Complex Proportional Assessment) methods, and we will use criteria like price, density and calorific power of coal. Thus we want to show that any of these two methods we can get the best alternative. As an alternatives we will use 4 types of coal: Kolubara, Kreka, Dried Vreoci and Banovici. The paper is organized as follows: In the second part the focus is on the SAW and COPRAS methods; the third section presents the numerical example that is based on the real data; followed by the conclusion in the fourth section.

2. SAW (SIMPLE ADDITIVE WEIGHTING) I COPRAS (COMPLEX PROPORTIONAL ASSESSMENT) METHOD

2.1. SAW(Simple Additive Weighting) method

SAW (Simple Additive Weighting) method is probably the best known and most used method of multicriterial analysis. It is a simple method, which often gives similar results to the so-called advanced method. It is directly applicable to the decision matrix, which consists these three steps:

- Normalization of decision matrix;
- Multiply the weighted normalized matrix coefficients;
- Addition of “difficult” parameter for each alternative.

Detailed procedures of the SAW method is presented below, and examples can be found in many papers

Step 1. Formation of normalized decision matrix $R=[r_{ij}]_{mxn}$. In the original version of the simple additive weight method we use a liner to transform the attribute values but there are many other approaches. Revenue for the attribute values r_{ij} is determined using the formula:

$$r_{ij} = \frac{x_j}{x_j^{\max}} \Big| j \in j^{\max}, i = 1,...m \qquad (1)$$

while for the expenditure we use formula:

$$r_{ij} = \frac{x_j}{x_j^{\min}} \Big| j \in j^{\min}, i = 1,...m \qquad (2)$$

where: x_{ij} is the ***i***-th performance alternative to the ***j***-th criterion/attribute, *m* is the number of alternatives, *n* is the number of criteria/attributes, j^{max} represents a set of revenue criteria/attributes, j^{min} represents a set of expenditure criteria/attributes, the $x_j^{\max}$ represents the maximum value of the *j*-th column of the matrix, which is determined using the formula:

$$x_j^{\max} = max\ x_{ij} \qquad (3)$$

$x_j^{\min}$ *r*epresents the minimum value of the *j*-th column of the matrix, which is determined using the formula:

$$x_j^{\min} = \min x_{ij} \qquad (4)$$

Step 2. Forming of the weighted normalized decision matrix $V=[v_{ij}]_{mxn}$. Weighted normalized value v_{ij} is calculated using the formula:

$$v_{ij} = w_j \cdot r_{ij},\ i = 1,...,m;\ j = 1,....,n \qquad (5)$$

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where w_j represents the weight/importance of the j-th criteria/attributes, and.....$\sum_{j=1}^{n} w_j = 1$

Step 3. Determination of total performance index of each alternative. Total (resulting/aggregate/ cumulative) performance index S_i is calculated using the following formula:

$$S_i=\sum_{j=1}^{n} v_{ij},\ i=1,...,m \quad (6)$$

Step 4. Choosing the best alternative or ranking of the alternatives. Alternatives considered are ranked in ascending order according to the value of S_i and the best alternative A* is determined using the following formula:

$$A^* \in \{A_i^* | = max\ S_i\}. \quad (7)$$

COPRAS (Complex Proportional Assessment) method

The detailed procedure of the COPRAS method is shown below:

Step 1. Formation of normalized decision matrix $R=[r_{ij}]_{mxn}$. Normalization of elements values of the decision matrix shall be the linear transformation-Sum method, where r_{ij} values are determined using the formula:

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}},\ i=1....,m;\ j=1,....,n \quad (8)$$

where: x_{ij} represents the performance of the i-th alternative to the j-th criterion/attribute, m is the number of alternatives and n is a number of criteria/attributes.

Step 2. Forming of the weighted normalized decision matrix $V=[v_{ij}]_{mxn}$. Weighted normalized value v_{ij} is calculated using the formula:

$$v_{ij} = w_j \cdot r_{ij}\ ,\ i=1,...,m;\ j=1,....,n \quad (9)$$

where w_j represents the weight/importance of the j-th criteria/attributes, and $\sum_{j=1}^{n} w_j=1$.

Step 3. Calculating the P_i and the R_i. P_i (Maximising indexes) and R_i (Minimizing indexes) are calculated using the formula:

$$P_i=\sum_{j=1}^{n} v_{ij}\ |\ j \in j^{max}\ ,\ i=1,...,m \quad (10)$$

$$R_i=\sum_{j=1}^{n} v_{ij}\ |\ j \in j^{min}\ ,\ i=1,...,m \quad (11)$$

where:

j^{max} represents a set of revenue criteria/attributes, and j^{min} a set of expenditure criteria/attributes.

Step 4. Determination of the relative importance (weight) for each alternative. The relative importance of alternatives Q_i is determined by the formula:

$$Q_i=1+\frac{\sum_{i=1}^{m} Ri}{Ri \sum_{i=1}^{m} \frac{1}{Ri}},\ i=1,....,m \quad (12)$$

Step 5. Choosing the best alternative or ranking of the alternatives. Alternatives considered are ranked in ascending order according to the value of Q_i and the best alternative A* is determined using the following formula:(Popovic et al., 2012).

$$A^* \in \{A_i^* | = max\ Q_i\}. \quad (13)$$

3. CHOICE OF COAL

Coal is black or brown-black sedimentary rock of organic origin which has the capability of burning and is used as a fossil fuel extracted from the ground with mining methods. It consists primary of carbon and hydrocarbons and other substances. It is an important fuel and source of electricity. It belongs to the non-renewable energy sources. There are various methods for the classification based on the origin, purpose, age, thermal power and other properties of the coal. In this paper we will compare the following types of coal: A1- Kolubara, A2- Kreka, A3- dried Vreoci and A4- Banovici. The question is which coal is the best alternative? There are many influences of the different criteria on the response to this question. We will take the 3 criteria and based on them we will choose the best alternative: C1- price, C2- calorific power, C3- density and C4- ash content.

3.1. Choosing the weight criteria using the AHP method

AHP method is used to determine the weight of criteria, which are as a result of consistency (*CR*) received ***CR=0,08*** which is less than ***0,1***, thus that the comparison is consistent.

3.2. Choosing the best alternative with the SAW method

Table 1 - Required data for selection of coal

CRITERIA/ATRIBUTES				
Name	Price	Calorific power	Density	Ash content
Measure unit	1000 RSD	MJ/kg	t/m^3	%
Weight	0,436	0,323	0,169	0,071
Optimization	Min.	Max.	Max.	Min.
	C1	**C2**	**C3**	**C4**
Alternative				
A1	6,5	8	1	14
A2	10,9	10,5	1,2	10
A3	12	12,5	1,15	18
A4	13,5	18,5	1,3	22

We will make the best choice of alternatives based on the following criteria:

- Price (**C1**): the price of coal, expressed in 1000 RSD
- Calorific value (**C2**): calorific value of coal, expressed in MJ/kg
- Density (**C3**): the density of coal, expressed in t/m^3
- The amount of ash (**C4**): ash content, expressed as %.

After eliminating data not necessary for the application of MDCM/MADM method we have the initial decision matrix, shown in table 2.

Table 2 - Initial decision matrix

Weight	0,44	0,32	0,17	0,07
Optimization	Min.	Max.	Max.	Min.
	C1	**C2**	**C3**	**C4**
Alternative				
A1	6,5	8	1	14
A2	10,9	10,5	1,2	10
A3	12	12,5	1,15	18
A4	13,5	18,5	1,3	22

Based on table 2. we form an appropriate decision matrix

$$D = \begin{array}{c} \\ \\ A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{array}{cccc} 0,44 & 0,32 & 0,17 & 0,07 \\ \text{min.} & \text{max.} & \text{max.} & \text{min.} \\ \end{array} \begin{bmatrix} 6,5 & 8 & 1 & 14 \\ 10,9 & 10,5 & 1,2 & 10 \\ 12 & 12,5 & 1,15 & 18 \\ 13,5 & 18,5 & 1,3 & 22 \end{bmatrix} \quad (14)$$

Linear normalization : type max

For a given decision matrix

$$D = \begin{array}{c} \\ \\ A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{array}{cccc} 0,44 & 0,32 & 0,17 & 0,07 \\ \text{min.} & \text{max.} & \text{max.} & \text{min.} \\ \end{array} \begin{bmatrix} 6,5 & 8 & 1 & 14 \\ 10,9 & 10,5 & 1,2 & 10 \\ 12 & 12,5 & 1,15 & 18 \\ 13,5 & 18,5 & 1,3 & 22 \end{bmatrix} \quad (15)$$

most acceptable alternative selection procedure using SAW method and linear normalization type max (LTmax), can be represented using the following steps:

Step 1. Formation of normalized decision matrix

$$R = \begin{bmatrix} 1 & 0,432 & 0,769 & 0,051 \\ 0,596 & 0,183 & 0,923 & 1 \\ 0,542 & 0,218 & 0,885 & 0,556 \\ 0,481 & 1 & 1 & 0,455 \end{bmatrix} \quad (16)$$

Step 2. Forming weighted normalized decision matrix

$$V = \begin{bmatrix} 1 & 0,432 & 0,769 & 0,714 \\ 0,596 & 0,568 & 0,923 & 1 \\ 0,542 & 0,676 & 0,885 & 0,556 \\ 0,481 & 1 & 1 & 0,455 \end{bmatrix} \text{X} \begin{bmatrix} 0,44 \\ 0,32 \\ 0,17 \\ 0,07 \end{bmatrix} = \begin{bmatrix} 0,436 & 0,140 & 0,130 & 0,051 \\ 0,260 & 0,183 & 0,156 & 0,071 \\ 0,236 & 0,218 & 0,150 & 0,039 \\ 0,210 & 0,323 & 0,169 & 0,032 \end{bmatrix} \quad (17)$$

Step 3. Calculate the resulting performance of each alternative and choosing the most appropriate one

$$S_i = \begin{array}{c} A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{bmatrix} 0,756 \\ 0,670 \\ 0,643 \\ 0,734 \end{bmatrix} \begin{array}{c} \leftarrow 1 \\ 3 \\ 4 \\ 2 \end{array} \quad (18)$$

Step 4. Based on the values of Si the most acceptable alternative was A1 (Kolubara coal).

3.3. The choice of the best alternative applying the COPRAS method

For a given decision matrix

$$D = \begin{array}{c} \\ \\ A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{array}{cccc} 0,44 & 0,32 & 0,17 & 0,07 \\ \text{min.} & \text{max.} & \text{max.} & \text{min.} \\ \end{array} \begin{bmatrix} 6,5 & 8 & 1 & 14 \\ 10,9 & 10,5 & 1,2 & 10 \\ 12 & 12,5 & 1,15 & 18 \\ 13,5 & 18,5 & 1,3 & 22 \end{bmatrix} \quad (19)$$

procedure of selecting the most acceptable alternative using the COPRAS methodwill be presented applying the following steps:

Step 1. Formation of normalized decision matrix

$$R = \begin{bmatrix} 0,152 & 0,162 & 0,215 & 0,219 \\ 0,254 & 0,212 & 0,258 & 0,156 \\ 0,280 & 0,253 & 0,247 & 0,281 \\ 0,315 & 0,374 & 0,280 & 0,344 \end{bmatrix} \quad (20)$$

Step 2. Forming weighted normalized decision matrix

$$V = \begin{bmatrix} 0,152 & 0,162 & 0,215 & 0,219 \\ 0,254 & 0,212 & 0,258 & 0,156 \\ 0,280 & 0,253 & 0,247 & 0,281 \\ 0,315 & 0,374 & 0,280 & 0,344 \end{bmatrix} \mathrm{X} \begin{bmatrix} 0,44 \\ 0,32 \\ 0,17 \\ 0,07 \end{bmatrix} = \begin{bmatrix} 0,066 & 0,052 & 0,036 & 0,016 \\ 0,111 & 0,069 & 0,044 & 0,011 \\ 0,122 & 0,082 & 0,042 & 0,020 \\ 0,137 & 0,121 & 0,047 & 0,024 \end{bmatrix} \quad (21)$$

Step 3. Determining the value of P and R

$$P = \begin{bmatrix} 0,089 \\ 0,112 \\ 0,123 \\ 0,168 \end{bmatrix} \quad (22)$$

$$R = \begin{bmatrix} 0,082 \\ 0,122 \\ 0,142 \\ 0,162 \end{bmatrix} \quad (23)$$

Step 4. Calculate the resulting performance of each alternative and choosing the most appropriate one

$$Q_i = \begin{matrix} A1 \\ A2 \\ A3 \\ A4 \end{matrix} \begin{bmatrix} 0,273 \\ 0,236 \\ 0,229 \\ 0,261 \end{bmatrix} \begin{matrix} \leftarrow 1 \\ 3 \\ 4 \\ 2 \end{matrix} \quad (24)$$

Step 5. Based on the values of Qi the most acceptable alternative was A1 (Kolubara coal).

CONCLUSION

Obtained results show that the Kolubara coal is the correct choice in the existing conditions. The following one is Banovici coal, third one is Kreka and on the last place is dried Vreoci. As previously stated, we were taken into account just some of the aspects such as: price, calorific power, density and ash content and the results were related exclusively to these criteria. The proposed methodology based on the SAW and COPRAS methods will assist in the selection of coal. Methodologies may include any number of criteria and offer objective, simpler and more consistent approach to the selection of coal. This methodology can be applied in the evaluation and ranking of different sets of alternative types of coal. Also, the choice of coal may be based on different criteria not only in this, that we used in our work.

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ИЗВОД

ВИШЕКРИТЕРИЈУМСКА АНАЛИЗА ИЗБОРА УГЉА ПРИМЕНОМ SAW И COPRAS МЕТОДА

Један од великих проблема како у сеоским тако и у урбаним срединама, јесте, да у време зимског периода изаберу најбољу алтернативу за грејање, конкретно угаљ. У раду ћемо приказати да применом SAW и COPRAS методама можемо уз одговарајуће критеријуме доћи до најбољих алтернатива. У раду је дат и нумерички пример који приказује примењљивост и ефективност предложених приступа.

Кључне речи: *избор угља, SAW, COPRAS*

Стручни рад

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